# Education and Borrowing Constraints: An Analysis of Alternative Allocation Systems

Raquel Fernández*
NYU, NBER and CEPR

Revised June 2008

**Abstract**

This paper compares the allocative properties of markets and exams in an environment in which students differ in wealth and ability and schools differ in quality. In the presence of borrowing constraints, exams are shown to dominate markets in terms of matching efficiency. Whether aggregate consumption is greater under exams than under markets depends on the power of the exam technology; for a sufficiently powerful test, exams dominate markets in terms of aggregate consumption as well. The effects of income taxation are analyzed and the optimal allocation scheme when wealth is observable is derived. The latter consists of allowing markets to set school prices but having the government allocate fellowships based both on financial need and exam score.

*I wish to thank Sherwin Rosen, Jonathan Portes, Paulo Medas, Jordi Gali, Alberto Bisin, for useful comments on a much earlier version of this paper as well as seminar participants at Princeton University, University of British Columbia, and the Board of Governors of the Federal Reserve. I am grateful to the NSF and to the C.V. Starr Center for Applied Economics for financial support.

*JEL Classification Nos.:* D52, E44, J41
*Keywords*: Education, Markets, Exams, Borrowing constraints, fellowships

# 1. Introduction

The provision of education contrasts sharply with the market mechanism used for other goods. In most instances, and at all levels of education, prices are not the sole mechanism used to allocate students to schools. Grades, examination results, extracurricular activities, etc. can all influence the admission decision. This is especially true at the secondary and tertiary levels of education, in which both prices and exams often play an important role in determining the allocation of students to schools.

The allocative role of prices in education is both direct, via a fee charged by a private school or university, or indirect, as when public education is locally provided. In the latter case, the price of purchasing or renting housing in a particular community (to obtain a desired level of spending on education or a specific composition of peers) affects allocations.[1] Exams are also used as allocative devices, both on their own to determine admissions at selective public secondary schools or universities, and in conjunction with prices at many private schools and universities. The relative importance of prices and exams in guiding allocation decisions varies across countries. In many European and Latin American countries, for examples, prices for tertiary education are very low and exams are used to determine admissions; private universities exist but are often considered second rate. In Japan, a one-shot entrance exam determines university admission and sorts students into their lifetime careers; rejection or acceptance by a prestigious university is seen as determining the student's entire future professional life. In the US, on the other hand, the majority of universities are private and both prices and exams play an important role in guiding the matching of students to universities.

Why is the allocation of education not done solely by prices? There are many reasons why a pure price mechanism may not optimal for education. It has been argued, for example, that education may have significant externalities, which may help explain why in

[1] This is the case in the US especially for primary and secondary education. Fernández and Rogerson (1996,1998), Benabou (1996), and Epple and Romano (1998), among others, analyze the local provision of education in a model with borrowing constraints.

all countries governments subsidize education and make some level of education mandatory. This does not in itself provide an explanation, however, for why exams are used to help guide the allocation decision. As long as the marginal return from investing in higher quality (or greater quantity) of education is larger for a higher-ability student, these students should be willing to outspend the lower ability students in order to attend higher-quality schools. This would result students sorting across schools by ability, with prices providing the correct signal with respect to which school each should attend. The role for government would be to subsidize lower-ability students to obtain education, or to encourage students of all abilities to attend school for a longer period of time than dictated strictly by their marginal private return.

Another argument against sole reliance on the price mechanism posits the existence of peer effects in education. However, this externality likewise does not in and of itself justify the existence of exams as an allocative mechanism. In an economy in which ability is perfectly observable and with perfect capital markets, prices alone would be sufficient to determine optimally the allocation of students to a given distribution of schools. This would entail charging students of different abilities different prices (including negative ones, i.e. paying some students to attend a particular school).[2] If ability is not observable (the more plausible assumption), then exams that give a signal about ability would be a useful additional instrument in solving the assignment problem.[3] In such a case schools would post prices based on exam score (i.e. ability type) but not based on financial status, i.e., students of the same ability should pay the same price at the same school or, to the extent that this is accomplished via fellowships, the latter should be based solely on merit.[4]

An important maintained assumption in the preceding discussion has been the existence

[2] See Rothschild and White (1995) for an analysis of this problem.

[3] In general, if ability is unobservable (and in the absence of exams), a competitive equilibrium may not exist (see Rothschild and Stiglitz (1976)).

[4] It is interesting to note that this is not what we observe in the majority of cases, at least for undergraduate education in the United States. The majority of scholarships have been based on financial need and independent of merit (once a cutoff level-admissions-has been reached). This is far less the case though in graduate education, where fellowships are awarded mainly on merit. As pointed out by a referee, undergraduate fellowships now often have a substantial merit component as well.

of perfect capital markets. Borrowing constraints, however, are often considered an important impediment to achieving a first-best allocation in a variety of environments.[5] This may be thought to be especially true in the case of education, where the unobservability of ability, moral hazard problems, and the fact that decisions are often made by parents rather than children, all serve to compound the problem of borrowing against future human capital.[6]

It may be thought that in the presence of borrowing constraints, an education system that offers access to schools of different qualities based on exam scores may have some advantages over one based solely on fees. There are several objections, however, that may be raised to such a view. In the first place, exams are not perfect indicators of ability. They are influenced by prior expenditures that to some extent augment human capital and to some extent merely enhance test performance (e.g. Stanley Kaplan courses to improves SAT scores).[7] Wouldn't then the competition for higher test scores run into the same borrowing constraint problems as under market prices, as wealthy individuals outspend poorer ones in an attempt to obtain higher exams scores? Furthermore, if these expenditures are wasteful (i.e., they do not enhance productivity directly), wouldn't they impose additional costs on an economy that would not be borne under market prices? The objective of this paper is to contrast the relative benefits of exams versus market prices in a simple model that is able to capture at least some of these concerns.

The paper starts by establishing a benchmark of equilibrium under perfect capital markets, contrasting the allocation obtained under markets with that obtained by exams. Exam scores are assumed to be affected (positively) both by ability and by expenditures. These

[5] Evans and Jovanovic (1989), for example, estimate that credit constraints are responsible for a total loss of investment by firms of 2.7 billion 1976 dollars annually and Pratap and Rendon (2003) estimate that over the period 1976-1995, the removal of liquidity constraints for a two digit industry could increase investment by an average of 34%.

[6] The potential importance of borrowing constraints in human capital accumulation has been studied in a variety of environments. See, for example, Becker and Tomes (1979), and Loury (1981), and more recently by Fernández and Rogerson (1998, 2003), Galor and Zeira (1993) and Glomm and Ravikumar (1992), among others. Fernández and Gali (1999) is the first paper to analyze the interaction of borrowing constraints and alternative allocation mechanisms such as exams, in the context of heterogeneous agents.

[7] Furthermore, it is often alleged that tests are culturally biased against some groups (racial, ethnic, gender, socio-economic, etc.).

expenditures are wasteful. That is, they do not in and of themselves increase utility nor output, but rather subtract from the resources available to an economy. Although both mechanisms achieve the efficient allocation, markets yield higher aggregate consumption due to the fact that exams involve wasteful expenditures.

Next the paper derives equilibrium under both allocation mechanisms under the assumption that individuals are unable to borrow. The interesting finding here is that exams always dominate markets with respect to matching efficiency, regardless of the exam's "power" to distinguish ability relative to expenditures.[8] Whether aggregate consumption is also higher under exams depends on the power of the latter; for a sufficiently powerful exam, aggregate consumption will indeed be higher as well. A subsequent section of the paper is devoted to an example that illustrates the properties of the two allocation mechanisms. The exam technology is parameterized such that the effect of different values of the "power" of an exam can be quantified.

The paper considers alternative policies (such as taxation) and identifies a mechanism that is able to achieve the first best when wealth is observable. This mechanism has the interesting feature of combining aspects of both market prices and exams, as is often observed in reality. However, perhaps counter to what intuition might suggest, these exams are not very demanding; they do not require individuals to obtain high scores. A last section of the paper concludes and makes suggestions for future research.

## 2. A Simple Model

This objective of this paper is to contrast the properties of market prices relative to exams in a simple model that allows the effects of alternative policies to be considered. An analysis of this problem was first done by Fernández and Gali (1999) in a model with a continuum of

[8] Freeman (1996) also obtains the result that exams can improve efficiency in a framework with identical agents and borrowing constraints. The reason for this though is very different from the one offered here. In his model even an exam that is a pure lottery increases efficiency by allowing agents to specialize and take advantage of increasing returns to scale.

school types and a uniform distribution of agents in wealth and ability. The analysis in that paper is far more complicated then the one presented here and they were unable to obtain an analytically tractable example. By contrast, the framework presented here consists of two types of schools and a general joint distribution of wealth and ability. The greater simplicity of the setting allows one to examine the effects of different interventions and of allocation mechanisms such as income taxation and fellowships.

Assume that there is a continuum of individuals, of measure one, characterized by their endowments of ability $a \epsilon [0,1] \equiv I$ and wealth $w \epsilon [0, \overline{w}] \equiv \Omega$. We start by assuming that both wealth and ability are unobservable, though nothing about how the two allocation mechanisms work depends on this.[9] The joint distribution of ability and wealth is given by $f(a, w)$, with $f(a, w) > 0$, $\forall a \epsilon I$, $\forall w \epsilon \Omega$. The cumulative distribution $F(a', w') = \int_0^{a'} \int_0^{w'} f(a, w)\, dwda$ is assumed to be continuous, with $F(a, 0) = 0$ and $F(1, \overline{w}) = 1$. Schools are in fixed supply and are distinguished by their quality, $s$, which can be either high $(H)$ or low $(L)$.[10] There is a measure one of schools and a fraction $1 - \alpha$ of them are of high quality. We assume that a measure $\mu$ of schools is able to accommodate exactly a measure $\mu$ of individuals (hence the high quality schools can accommodate a fraction $1 - \alpha$ of the population).

An individual with ability $a$ who attends school $s$, $s \in \mathrm{S} = \{L, H\}$, obtains $X(a, s)$, where $X$ can be thought of as a production function or as a human capital function (with the price of a unit of human capital or of the output generated normalized to equal one). We assume that agents have strictly increasing utility functions over consumption and that their output from not attending any school is normalized to zero. Furthermore, we assume $X$ is continuous, bounded, with $X_a \geq 0$, $X_s \geq 0$ (i.e., output or human capital is increasing both in ability and schooling), and $X_{as} > 0$, i.e., ability and schooling are complements. This last assumption plays a key role in the characterization of efficient allocations. To facilitate exposition, assume $X(0, L) = 0$, (i.e., the lowest ability agent obtains a payoff of zero when

[9] Observability of ability renders the mechanism design problem unintersting since a social planner could simply mandate which individuals should attend which schools. Section 5 examines a fellowship scheme under the assumption that wealth is observable.

[10] Alternatively, one can interpret the low-quality school as the payoff to not attending school at all.

she attends the lower-quality school).[11]

Independently of the allocation mechanism and borrowing environment, the actions of agents can be thought of as occurring within the following two-stage framework. In the first stage, individuals incur their desired expenditures (and desired borrowing as well if capital markets are operating) and are allocated, according to the assignment rules, to a school $S(a, w)$ ($S : I\text{x}\Omega \rightarrow \text{S}$). In the second stage they generate output $X$, repay loans (if any), and consume their output plus initial wealth minus debt repayment and first-stage expenditures. Note that the distribution of schools and individuals is exogenous and that there are no peer effects or externalities.

### 2.1. The Efficient Allocation

Before comparing allocation mechanisms, it is useful to review the efficient allocation in this framework. The assumptions on technology (namely $X_{as} > 0$) render the matching problem in our model identical to Becker's (1973) marriage problem.[12] Thus, in our two-school and continuum-of-agent-types model, the efficient allocation is characterized by a partition of the ability space such that all individuals with $a > a^*$ are assigned to $H$ and all those with $a < a^*$ are assigned to $L$, independently of their wealth. The cutoff level $a^*$ exactly exhausts the capacity of the schools, i.e., $F(a^*, \overline{w}) = \alpha$. This result follows entirely from the assumption of complementarity between $a$ and $s$, i.e. from $X_{as} > 0$.

### 2.2. Two Allocation Mechanisms

We consider two different environments with two different assignment rules: (i) a private market that allocates individuals to schools based on their willingness and ability to pay a school's price and (ii) public ownership of schools that allocates individuals to schools based

[11] As will be clear further on, this assumption allows us to get rid of price equilibria under perfect capital markets that differ from each other solely by a constant.

[12] Thus, our paper can also be seen as contributing to a relatively small literature that examines matching problems, including Cole, Mailath, and Postlewaite (1992), Acemoglu (1997), Kremer and Maskin (1996), Legros and Newman (2002), and Burdett and Coles (1996), by analyzing the effect of borrowing constraints. See Sattinger (1993) for a review of assignment models.

on their exam performance, assigning those individuals who obtain test scores over some cutoff level to $H$, and all others to $L$.

Before comparing some of the general properties of the two allocation mechanisms, we need to present additional features of exams. The exam is a technology $V$ whose outcome (the score) - $v$ - depends both on ability and on score-enhancing expenditures, $e$. Thus, $v = V(a, e)$ and we assume $V_a > 0$, $V_e > 0$ with $V(a, 0) = 0$, $\forall a \in I$ (i.e. expenditures are necessary to obtain a positive score).[13] These expenditures can be thought of as being incurred directly in exam preparation (tutors, etc.) or indirectly via the choice of elementary and secondary schools (if these are tertiary-level schools).

It is often convenient to work with the dual of $V$, namely $E(a, v)$. This gives the expenditures needed by an agent of ability $a$ to obtain a score of $v$. Note that our assumptions on $V$ imply $E_a < 0$ and $E_v > 0$. We also assume $E_{va} \leq 0$ (that is, the change in expenditures needed to obtain a marginal increase in a given score is non-increasing in $a$).[14]

The assumptions on technology imply that both mechanisms induce similar preferences over outcomes on the part of individuals. In particular, in both environments the indifference curves of individuals are single crossing. A higher-ability individual is willing to suffer a greater price increase or critical score increase in order to attend a higher-quality school than is a lower-ability individual.[15]

Single-crossing indifference curves have powerful implications for the properties of equilibrium. Under perfect capital markets, this property implies that individuals with greater ability will always be willing (and able) to outspend or outscore lower-ability individuals to ensure that they are allocated to $H$, i.e., $S(a'', w'') \geq S(a', w')$; $\forall a'' > a'$, $\forall w$. If individuals face borrowing constraints, then under a market mechanism this property instead implies $S(a'', w'') \geq S(a', w')$; $\forall a'' > a'$ and $\forall w'' \geq w'$ (indicating that individuals of greater ability

[13] This last assumption is not essential; it is made to facilitate characterization of equilibrium.

[14] This assumption ensures that indifference curves are single crossing, i.e., $\frac{\partial}{\partial a}(\frac{dv}{ds}\mid_{u=\overline{u}}) = \frac{X_{as}E_v - E_{va}X_s}{E_v^2} > 0$.

[15] An alternative interpretation of the exam technology is that the score depends on effort and expenditures and that the marginal (utility) cost effort depends on ability. One can then impose the appropriate conditions on the preferences induced by this technology to obtain single-crossing.

will always be allocated to at least as high an $s$ if they are at least as wealthy). With exams, the corresponding implication is $S(a'', w'') \geq S(a', w')$; $\forall a'' > a'$ and $\forall w', w''$ such that $V(a'', w'') \geq V(a', w')$ i.e., individuals of higher ability will be allocated to at least as high an $s$ if they can afford to generate at least the same score.

We will be interested in comparing aggregate output consumption under both mechanisms. In order to do this, we need to discuss how expenditures made under exams compare with fees charged by schools under markets. We assume that expenditures to improve exam scores are completely wasteful in the sense that they do not in and of themselves contribute to consumption or utility. Hence one can think of those expenditures as withdrawing resources (say tutors for SAT exams) from other productive potential uses in the economy, without in and of themselves increasing output.[16] Fees charged for schools under markets, on the other hand, are simply a transfer from one set of individuals to another (from students to the owners of schools). Although without lump-sum transfers among agents we cannot speak of one mechanism Pareto dominating another, for the sake of comparison we evaluate two features: (i) Matching efficiency (i.e. aggregate output), and (ii) Aggregate consumption (aggregate output plus aggregate wealth minus any exam expenditures).

In what follows, $*$ denotes variables under perfect capital markets and $\widehat{}$ is used for variables under borrowing constraints. A subscript of $M$ denotes the market mechanism; one of $T$ denotes the test or exam mechanism.

## 3. The Market Equilibrium

We start by assuming that agents have access to perfect capital markets. In order to avoid the (unenlightening in this case) complications that accompany endogenizing the market for

[16] It might well be argued that going to, say, a better high school, produces greater human capital as well as enhancing exam taking technique, so that both output and the exam technology are affected by greater expenditures, i.e. $X(a+h(a,e), s)$ and $v = V(a+h(a,e), e)$ where $h$ is the additional human capital produced by an agent of ability $a$ and expenditures of $e$. As long as both under markets and exams we still obtain single-crossing indifference curves (which under reasonable assumptions for $h$ we do), then the same sorting implications obtain. Considering all expenditures as wasteful only makes the case for exams harder.

loans, we assume that this is an external capital market that operates at a constant and riskless interest rate which without loss of generality is normalized to zero. We next turn to characterizing the market equilibrium under perfect capital markets and with borrowing constraints.

### 3.1. The Market Mechanism with Perfect Capital Markets

An equilibrium is (*i*) a price for each school such that demand for schools of each type equals supply, (*ii*) schools maximize profits (i.e. given that the distribution of schools is in fixed supply, in this context they simply charge the greatest price they can), and (*iii*) individuals maximize utility.

Given that schools are infinitesimally small and hence perfectly competitive, schools of the same quality must charge the same price in equilibrium. With perfect capital markets, higher-ability agents are willing and able to outspend lower ability agents to attend a higher quality school. Thus, the agents that attend $L$ must include those with ability zero. Consequently, the assumptions of $X(0, L) = 0$, $f(a, w) > 0$, and continuity of $X$ with respect to $a$ imply that the price of a low-quality school, $P_L$, equals zero.

Single-crossing implies that equilibrium can be represented by an ability and high-quality school price pair $(a_M^*, P_H^*)$ such that all individuals with ability lower than $a_M^*$ attend $L$ and those with higher ability attend $H$. Utility maximization implies that all individuals must weakly prefer their allocation to the alternative. Hence, for all $(a, w)$ such that $S(a, w) = H$, given $P_L = 0$, we must have $\Delta(a) \geq P_H^*$, where

$$\Delta(a) \equiv X(a, H) - X(a, L) \tag{3.1}$$

(with the reverse inequality holding for those who attend $L$).[17] Furthermore, the fraction of individuals that attend $H$ must equal $1 - \alpha$. This, and continuity of $X$ with respect to $a$,

[17] Note that single-crossing implies $\Delta(a)$ increasing in $a$.

yields the following two equilibrium conditions for $(a_M^*, P_H^*)$:

$$\Delta(a_M^*) = P_H^* \tag{3.2}$$

$$\int_{a_M^*}^{1} \int_0^{\overline{w}} f(a,w)\ dw\ da = 1 - \alpha \tag{3.3}$$

with single-crossing implying:

$$S(a,w) = \begin{cases} H,\ \forall a \geq a_M^* \\ L,\ otherwise \end{cases} \tag{3.4}$$

Note that $P_H^*$ exists and is unique since the fraction of individuals for whom $\Delta(a) \geq P_H$ is continuous and decreasing in $P_H$ (with a fraction of one for whom $\Delta(a) \geq P_H = 0$ and a fraction of zero for whom $\Delta(a) \geq P_H = \Delta(1)$.

### 3.2. The Market Mechanism with Borrowing Constraints

In this section we assume that all individuals are unable to borrow. We do not model here the reason behind the market failure, though, for example, inability to penalize recalcitrant borrowers or an unverifiable output level would be sufficient to close down capital market.

An equilibrium is now a pair $(\widehat{a}_M, \widehat{P}_H)$ such that $(i)$ individuals maximize their utility subject to a no-borrowing constraint, $(ii)$ schools maximize profits, and $(iii)$ demand for schools of each type equals their supply.

To begin with, note again that the same assumptions as before imply that the equilibrium price of a low-quality school is zero. Consequently, $(\widehat{a}_M, \widehat{P}_H)$ must satisfy:

$$\Delta(\widehat{a}_M) = \widehat{P}_H \tag{3.5}$$

$$\int_{\widehat{P}_H}^{\overline{w}} \int_{\widehat{a}_M}^{1} f(a,w)\ da\ dw = 1 - \alpha \tag{3.6}$$

with single crossing implying

$$S(a,w) = \begin{cases} H, \forall a \geq \widehat{a}_M \text{ and } w \geq \widehat{P}_H \\ L, \, otherwise \end{cases} \quad (3.7)$$

Equilibrium can now be depicted as in Figure 1 with all the agents in the shaded area attending $H$ and all others attending $L$. Existence and uniqueness of equilibrium can be proved as under perfect capital markets.

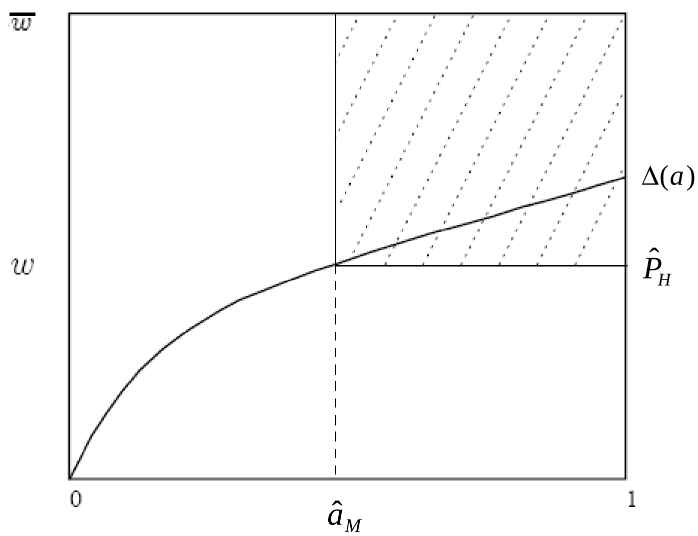


Figure 1

### 3.3. A Comparison of Markets With and Without Borrowing Constraints

It is of interest to compare the functioning of markets with and without borrowing constraints. A first result is that the lowest ability individual to attend $H$ in equilibrium is higher under perfect capital markets than under borrowing constraints, i.e.,

**Theorem 3.1.** $\widehat{a}_M < a^*$.

*Proof*: With perfect capital markets, we have $1-\alpha = \int_0^{\overline{w}} \int_{a^*}^1 f(a,w)dadw > \int_{\Delta(\widehat{a}_M)}^{\overline{w}} \int_{a^*}^1 f(a,w)\, dadw$, implying $\widehat{a}_M < a^*$.||

It is now easy to show that the price of high-quality schooling is higher under perfect capital markets than under borrowing constraints.

**Corollary 3.2.** $P_H^* > \widehat{P}_H$.

*Proof*: This follows directly from Theorem (3.1) and equations (3.2) and (4.4).||

Neither of the results above are surprising: borrowing constraints require a lower price for the high-quality school in order to fill capacity in both types of schools. But this decreased price makes the high-quality school more attractive to lower ability agents, decreasing the ability associated with the "marginal" agent that attends $H$ relative to perfect capital markets.

Of course, the change in allocation implies that output (and hence consumption) is also lower under borrowing constraints relative to perfect capital markets, i.e.,

$$\widehat{Y}_M + W = \widehat{C}_M < Y^* + W = C^* \tag{3.8}$$

where

$$\begin{aligned} \widehat{Y}_M \quad = \quad & \int_{\widehat{a}_M}^{1} \int_{\widehat{P}_H}^{\overline{w}} X(a,H) f(a,w)\ dw\ da + \\ & \int_{0}^{\widehat{a}_M} \int_{\widehat{P}_H}^{\overline{w}} X(a,L) f(a,w)\ dw\ da + \int_{\widehat{a}_M}^{1} \int_{0}^{\widehat{P}_H} X(a,L) f(a,w) dw\ da \end{aligned} \tag{3.9}$$

$$W \equiv \int_0^1 \int_0^{\overline{w}} w f(a,w)\ dwda.$$

and

$$Y^* = \int_{a^*}^{1} \int_{0}^{\overline{w}} X(a,H) f(a,w) dwda + \int_{0}^{a^*} \int_{0}^{\overline{w}} X(a,L) f(a,w) dwda \tag{3.10}$$

is the aggregate output obtained under efficient matching and hence also with the market mechanism under perfect capital markets.

## 4. The Exam Equilibrium

The assignment mechanism under exams is a rule that states a cutoff score level $v_c$ such that all individuals with $v \geq v_c$ attend $H$ and all others attend $L$. Since schools in this environment are completely passive and do not obtain any revenue (i.e., do not charge a price), it is best to think of them as being owned by the state, with the government setting $v_c$ such that $1-\alpha$ agents obtain at least this score. Note that since higher scores mean greater expenditures, no agent will in equilibrium obtain a score greater than $v_c$. Furthermore, those agents who in equilibrium attend $L$ will not expend any resources in the generation of scores. Hence, an equilibrium is $(i)$ a score $v_c$ such that a fraction $1-\alpha$ of agents obtain that score, and $(ii)$ individuals maximize utility.

### 4.1. The Exam Mechanism with Perfect Capital Markets

As with the market mechanism, single crossing implies that equilibrium can be characterized by a pair $(a_T^*, v^*)$ such that all individuals with ability greater than $a_T^*$ obtain the score $v^*$ and attend $H$. Utility maximization implies that these individuals must be at least as well off as by making zero expenditures and attending $L$. Hence, for these individuals it must be the case that $\Delta(a) \geq E(v^*, a)$ (with the opposite inequality holding for those agents that attend $L$). Consequently, equilibrium conditions on $(a_T^*, v^*)$ require:

$$\Delta(a_T^*) = E(a_T^*, v^*) \tag{4.1}$$

$$\int_{a_T^*}^{1} \int_{0}^{\overline{w}} f(a, w)\ dw\ da = 1 - \alpha \tag{4.2}$$

with single-crossing implying:

$$S(a, w) = \begin{cases} H,\ \forall a \geq a_T^* \\ L,\ otherwise \end{cases} \tag{4.3}$$

Note that $(a_T^*, v^*)$ exists and is unique since the fraction of agents with $\Delta(a) \geq E(a, v)$ is continuous and decreasing in $v$ (with a fraction of one such that $\Delta(a) \geq E(a, 0)$ and a fraction of zero for whom $\Delta(a) \geq E(a, \overline{v})$, where $\overline{v}$ is defined by $E(1, \overline{v}) = \Delta(1)$).

### 4.2. The Exam Mechanism with Borrowing Constraints

With borrowing constraints, an equilibrium under exams is an $(\widehat{a}_T, \widehat{v})$ pair such that $(i)$ individuals maximize utility subject to a no-borrowing constraint and $(ii)$ a fraction $1 - \alpha$ of agents obtain a score of $\widehat{v}$. Thus, $(\widehat{a}_T, \widehat{v})$ must satisfy:

$$\Delta(\widehat{a}_T) = E(\widehat{a}_T, \widehat{v}) \tag{4.4}$$

$$\int_{\widehat{a}_T}^{1} \int_{E(a,\hat{v})}^{\overline{w}} f(a, w)\; dw\; da = 1 - \alpha \tag{4.5}$$

with

$$S(a, w) = \begin{cases} H,\ \forall a \geq \widehat{a}_T \text{ and } w \geq E(a, \widehat{v}) \\ L,\ otherwise \end{cases} \tag{4.6}$$

Equilibrium is depicted in Figure 2. The shaded area represents the set of agents that in equilibrium attend $H$. Existence and uniqueness of equilibrium can be proved as under perfect capital markets.

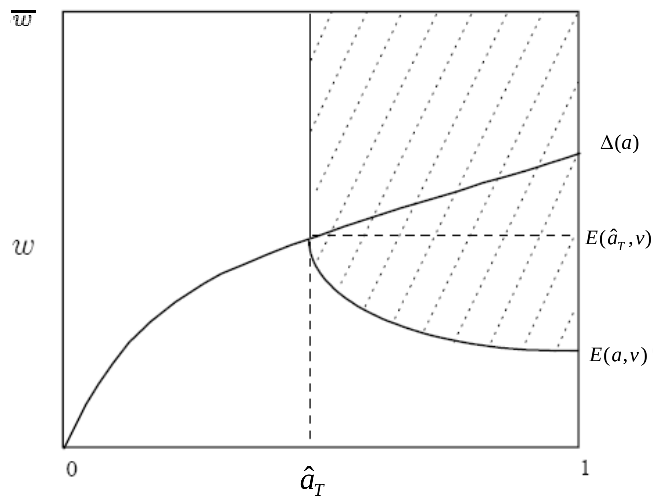


Figure 2

### 4.3. A Comparison of Exams With and Without Borrowing Constraints

As with markets, it is easy to show that the effect of borrowing constraints is to allow lower-ability level agents to attend $H$, and to decrease the score associated with the latter.

**Theorem 4.1.** $\widehat{a}_T < a^*$ .

*Proof*: Noting that $\widehat{v} > 0$, we have $1-\alpha = \int_{a^*}^{1} \int_{0}^{\overline{w}} f(a,w)\, dw\, da > \int_{a^*}^{1} \int_{E(a,\widehat{v})}^{\overline{w}} f(a,w)\, dw\, da$. Hence, $a^* > \widehat{a}_T$.||

**Corollary 4.2.** $\widehat{v} < v^*$ .

*Proof*: Follows immediately from Theorem 4.1 and equations (4.1) and (4.4).||

As with our comparison of markets with and without borrowing constraints, obtaining a different allocation with borrowing constraints than that obtained under perfect capital markets implies $Y^* > \widehat{Y}_T$ where

$$\widehat{Y}_T = \int_{\widehat{a}_T}^{1}\int_{E(a,\widehat{v})}^{\overline{w}} X(a,H)f(a,w)dwda + \int_{\widehat{a}_T}^{1}\int_{0}^{\overline{w}} X(a,L)f(a,w)dwda + \int_{\widehat{a}_T}^{1}\int_{0}^{E(a,\widehat{v})} X(a,L)f(a,w)\ dwda \tag{4.7}$$

Furthermore,

$$\widehat{C}_T = \widehat{Y}_T + W - \widehat{\varepsilon} \tag{4.8}$$

where

$$\widehat{\varepsilon} = \int_{\widehat{a}_T}^{1}\int_{E(a,\widehat{v})}^{\overline{w}} E(a,\widehat{v})\ dwda \tag{4.9}$$

This does not allow us to conclude, however, that with the exam mechanism consumption under perfect capital markets, $C_T^*$ is greater than $\widehat{C}_T$, since if wasteful expenditure under perfect capital markets is sufficiently larger than that under borrowing constraints, aggregate consumption can actually be greater under the latter.

## 5. Markets vs Exams

We next turn to a comparison of markets and exams. The first thing to note, comparing equations (3.3) and (4.2), is that with perfect capital markets both mechanisms achieve the same final efficient allocation, i.e.

$$a_M^* = a_T^* = a^* \tag{5.1}$$

implying that aggregate output is likewise the same, i.e., $Y_M^* = Y_T^* = Y^*$. Aggregate consumption, on the other hand, differs as a result of the sum of wasteful expenditures, $\varepsilon^*$, with exams. Hence,

$$C_T^* = Y_T^* + W - \varepsilon^* = C_M^* - \varepsilon^* = C^* - \varepsilon^* \tag{5.2}$$

where

$$\varepsilon^* = \int_{a^*}^{1} \int_{0}^{\overline{w}} E(a, v^*) f(a, w) \; dw da \tag{5.3}$$

Lastly, note that equations (5.1), (3.2) and (4.1) imply that

$$P_H^* = E(a^*, v^*) \tag{5.4}$$

i.e. the price of the high quality school under markets equals the exam expenditures of the lowest ability agent to attend $H$ in equilibrium.

The ability of exams to achieve matching efficiency is due to the assumptions on technology; the exam technology does not distort higher-ability agents' willingness to outscore lower-ability ones, and perfect capital markets make it feasible for them to do so. The lower aggregate consumption achieved under exams follows immediately from our assumption of exam expenditures being wasteful.

Next we compare the performance of the exam and market mechanisms with borrowing constraints. We start out by showing that the lowest-ability individual to attend $H$ under exams is of higher ability than the lowest-ability individual to attend $H$ under markets.

**Theorem 5.1.** $\widehat{a}_T > \widehat{a}_M$ .

*Proof*: Suppose not and instead suppose $\widehat{a}_T \leq \widehat{a}_M$. By equations (4.4) and (3.5), this implies $\Delta(\widehat{a}_T) \leq \Delta(\widehat{a}_M)$ and hence $E(\widehat{a}_T, \widehat{v}) \leq \widehat{P}_H$. But, $1 - \alpha = \int_{\widehat{a}_T}^{1} \int_{E(a,\widehat{v})}^{\overline{w}} f(a, w) dw da > \int_{\widehat{a}_T}^{1} \int_{E(\widehat{a}_T,\widehat{v})}^{\overline{w}} f(a, w) dw da \geq \int_{\widehat{a}_M}^{1} \int_{\widehat{P}_H}^{\overline{w}} f(a, w) dw da = 1 - \alpha$, a contradiction.||

**Corollary 5.2.** $E(\widehat{a}_T, \widehat{v}) > \widehat{P}_H$.

*Proof*: Follows immediately from the previous theorem and equations (4.4) and (3.5).||

An important implication of the preceding results is that, with borrowing constraints, exams possess greater matching efficiency than markets. This is depicted in Figure 3 which compares the school assignments under exams and markets. Note that the area contained to the North-East of $ABC$ represents the set of agents that attend $H$ under exams and the area $DE\widehat{P}_H$ does the same for markets. The complements of each of these areas represents the set of agents that attend $L$ under exams and markets respectively. Thus all agents in the shaded areas have the same assignments under both mechanisms. The agents in $ABFED$ (hereafter area 2), however, attend $H$ under markets and $L$ under exams, whereas the agents represented by the area $FC\widehat{P}_H$ (hereafter area 1) attend $L$ under markets and $H$ under exams. Given the fixed capacity of the school system, areas 1 and 2 must contain the same measure of agents. But, then, $X_{as} > 0$ implies that the net gain from reallocating higher ability agents to higher-quality schools in exchange for lower ability agents to lower-quality schools is positive, i.e., $\int\int_1 (X(a,H)-X(a,L))f(a,w)dwda > \int\int_2 (X(a,H)-X(a,L))f(a,w)\,dwda$.

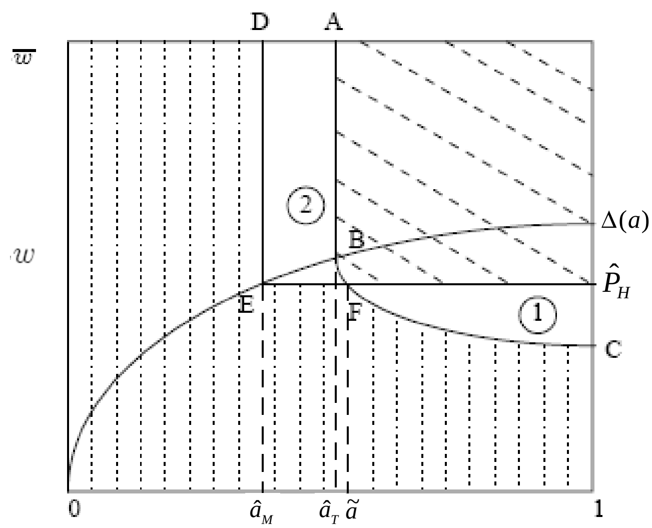


Figure 3

**Theorem 5.3.** $\widehat{Y}_T > \widehat{Y}_M$ .

*Proof*: Follows immediately from above. ||

Contrary to what intuition may suggest, the greater aggregate output obtained under exams does not result from the "marginal price" of the high-quality school being lower under exams than under markets. Indeed, as Corollary 5.2 establishes, the opposite is true, i.e., $E(\widehat{a}_T, \widehat{v}) > \widehat{P}_H$. Rather, the more efficient matching is due to the lower effective price for sufficiently high ability individuals, i.e., to the properties of the test technology, which make it less expensive for higher ability individuals to obtain a given score and more expensive for lower ability agents to do so. Exams make it possible for individuals to use another attribute–ability–to compete, which serves to effectively loosen borrowing constraints for high ability individuals and tighten them for low ability individuals.

Of course, a comparison of matching or production efficiency of two allocation mechanisms is not as interesting as a comparison of the consumption they permit, since the magnitude of the resources expended in achieving the final allocation matters. Exams do not necessarily dominate markets in terms of aggregate consumption. In general, which mechanism is superior in this regards depends upon the "power" of the testing technology, the production function, and the joint income and ability distribution of agents. Below I establish a limiting result that guarantees that a sufficiently "powerful" testing technology (in a sense that will be made rigorous) implies that exams will dominate markets in terms of aggregate consumption as well.

**Theorem 5.4.** *Given a test technology that can be written as* $E(a, v) = h(v)g(a)^{-\lambda}$ *with* $h(0) = 0, h' > 0$ *and* $g, g' > 0$*, we can define a family of technologies,* $E_\lambda$*, indexed by the value of* $\lambda$*,* $\lambda > 0$*. Then,* $(i)$ $\lim_{\lambda\to\infty} E_\lambda = 0$*,* $\forall a > a^*$*, and* $(ii)$ $\lim_{\lambda\to\infty} \widehat{a}_T(\lambda) = a^*$*.*

*Proof*: We start out by showing that $\forall a > a^* + \epsilon$, $\epsilon > 0$, $\lim_{\lambda\to\infty} E_\lambda(a, \widehat{v}(\lambda)) = 0$. Recall that, by equation (4.4), $E_\lambda(\widehat{a}_T(\lambda), \widehat{v}(\lambda)) = \Delta(\widehat{a}_T(\lambda))$. Hence, $h(\widehat{v}(\lambda)) = \Delta(\widehat{a}_T(\lambda))g(\widehat{a}_T(\lambda))^\lambda$ and for all $a > \widehat{a}_T$, $E_\lambda(a, \widehat{v}(\lambda)) = \Delta(\widehat{a}_T(\lambda)) \left[\frac{g(\widehat{a}_T)}{g(a)}\right]^\lambda$. Note that $\Delta(\widehat{a}_T(\lambda))$ is bounded above by $\Delta(a^*)$ and below by $\Delta(\widehat{a}_M)$. Note furthermore that $\forall a > a^* + \epsilon$, $\epsilon > 0$, $\frac{g(\widehat{a}_T)}{g(a)}$ is smaller than one and is bounded from above by $\frac{g(a^*)}{g(a^*+\epsilon)}$. Hence, $\lim_{\lambda\to\infty} E_\lambda(a, \widehat{v}(\lambda)) = 0$, $\forall a > a^* + \epsilon$.

Note that the above implies that all individuals with $a > a^* + \epsilon$ attend $H$, since they can afford to and furthermore prefer it to $L$. But, since this conclusion holds for all $\epsilon > 0$, it follows that $\lim_{\lambda\to\infty}\widehat{a}_T(\lambda) = a^*$.||

**Corollary 5.5.** $\lim_{\lambda\to\infty} \widehat{C}_T(\lambda) = C^* > \widehat{C}_M$.

*Proof*: The preceding theorem established that as $\lambda$ tends to $\infty$, the allocation of individuals to school tends to the efficient allocation and test expenditures tend to zero. Hence aggregate consumption tends to its first best level, $C^*$.||

The above theorem establishes that with a sufficiently powerful test technology exams dominate markets both in terms of matching efficiency and in terms of aggregate consumption. In the limit, matching is efficient, guaranteeing that output is maximized. Furthermore, in the limit waste is zero, so aggregate consumption is also at its maximum.

The intuition for the above result comes from noting that an increase in $\lambda$ decreases $E_a$. Noting that $E_a = -\frac{V_a}{V_e}$, a higher $\lambda$ increases the importance of ability relative to expenditures. That is, ceteris paribus, an increase in $\lambda$ makes it easier for higher ability but lower wealth individuals to compete with lower ability but higher wealth individuals with respect to admission to a high quality school. And, precisely because, ceteris paribus, a greater proportion of the competition can come through ability than through expenditures, waste may be reduced.[18]

It should be noted that the opposite result to that of Corollary 5.5 is obtained if we instead take the limit as $\lambda \to 0$. This makes ability totally irrelevant to the score obtained, i.e. only expenditures matter. In that case, the allocation achieved under exams is the same as under markets. Under exams, wasteful spending of the amount $(1-\alpha)\widehat{P}_H$ occurs, however, guaranteeing that while output is the same under both mechanisms, exams achieve lower aggregate consumption.

## 6. An Example

This section provides an example that illustrates how exams and markets fare as allocation mechanisms. We consider the following case: individuals are distributed uniformly in both $a$ and $w$; $X(a, s) = as$, hence $\Delta(a) = az$, $z \equiv H - L$; and, lastly, $E(a, v) = h(v)a^{-\lambda}$ (with

[18] The reason for the ceteris paribus qualifier in the statement is that a more powerful technology will increase the value of $\widehat{a}_T$ but this will also increase the expenditures of this agent and of agents with similar ability levels. Thus, waste may not decrease monotonically with $\lambda$.

$h, h' > 0$).

We start out by contrasting the equilibrium achieved under both mechanisms with perfect capital markets. Note that the uniform distribution of individuals implies $a^* = \alpha$ and $W = \frac{\overline{w}}{2}$.

Aggregate output is given by:

$$\begin{aligned} Y^* &= \frac{1}{\overline{w}} \int_{a^*}^{1} \int_{0}^{\overline{w}} aH \; dwda + \frac{1}{\overline{w}} \int_{0}^{a^*} \int_{0}^{\overline{w}} aL \; dwda \\ &= \frac{H(1-\alpha^2)}{2} + \frac{L\alpha^2}{2} \end{aligned} \tag{6.1}$$

and thus, $C^* = C_M^* = Y^* + \frac{\overline{w}}{2}$.

To calculate consumption under exams, note that $\Delta(\alpha) = E(\alpha, v^*)$, yielding $h(v^*) = z\alpha^{1+\lambda}$. Thus,

$$\varepsilon^* = \frac{1}{\overline{w}} \int_{a^*}^{1} \int_{0}^{\overline{w}} h(v^*) a^{-\lambda} \; dwda = \frac{z\alpha^{1+\lambda}(1-\alpha^{1-\lambda})}{1-\lambda} \tag{6.2}$$

and

$$C_T^* = Y^* - \varepsilon^* + W = \frac{H(1-\alpha^2)}{2} + \frac{L\alpha^2}{2} - \frac{z\alpha^{1+\lambda}(1-\alpha^{1-\lambda})}{1-\lambda} + \frac{\overline{w}}{2} \tag{6.3}$$

With borrowing constraints, using equilibrium conditions (3.5) and (3.6), yields:

$$\widehat{a}_M = \frac{(z+\overline{w}) - \sqrt{(z+\overline{w})^2 - 4z\alpha\overline{w}}}{2z} \tag{6.4}$$

and using equation (3.9) gives:

$$\widehat{Y}_M = \frac{1}{2\overline{w}} [H(1-\widehat{a}_M^2)\overline{w} + L\widehat{a}_M^2(\overline{w} - z\widehat{a}_M)] - z^2\widehat{a}_M \left(1 - \widehat{a}_M^2\right) \tag{6.5}$$

with $\widehat{C}_M = \widehat{Y}_M + \frac{\overline{w}}{2}$.

Under exams, applying equations (4.4) and (4.5) we obtain an implicit expression for $\widehat{a}_T$:

$$\frac{1}{\overline{w}} \int_{\widehat{a}_T}^{1} \int_{z\widehat{a}_T\left(\frac{\widehat{a}_T}{a}\right)^{\lambda}}^{\overline{w}} dw \; da = 1 - \alpha \tag{6.6}$$

and, using equations (4.7) and (4.9), yields:

$$\widehat{Y}_T = \frac{H(1-\widehat{a}_T^2)}{2} - \frac{zH\widehat{a}_T^{1+\lambda}(1-\widehat{a}_T^{2-\lambda})}{\overline{w}(2-\lambda)} + \frac{L\widehat{a}_T^2}{2} + \frac{zL\widehat{a}_T^{1+\lambda}(1-\widehat{a}_T^{2-\lambda})}{\overline{w}(2-\lambda)} \tag{6.7}$$

$$\widehat{\varepsilon} = \frac{z\widehat{a}_T^{1+\lambda}(1-\widehat{a}_T^{1-\lambda})}{1-\lambda} - \frac{z^2\widehat{a}_T^{2(1+\lambda)}(1-\widehat{a}_T^{1-2\lambda})}{(1-2\lambda)\overline{w}} \tag{6.8}$$

with $\widehat{C}_T$ given by $\widehat{Y}_T - \widehat{\varepsilon} + \frac{\overline{w}}{2}$.

Tables 1 and 2 report the equilibrium values of $a$, output, consumption (net of aggregate wealth), and in the case of exams, wasteful expenditures, under perfect capital markets and with borrowing constraints respectively, for different values of $\lambda$. Under perfect capital markets, the values of output and $a^*$ are invariant to the value of $\lambda$ since the efficient allocation (and hence output) is independent of the test technology. Consumption under markets is likewise independent of $\lambda$ since outcomes under markets do not depend on the test technology. With exams, note that the value of waste, $\varepsilon^*$, and hence consumption, $c^*$, is affected by $\lambda$, with waste decreasing, and hence consumption increasing, for greater values of $\lambda$.

Under borrowing constraints, for $\lambda$ relatively small, markets dominate exams in terms of aggregate consumption. At an intermediate value of $\lambda$ ($\lambda = 5.309$), both mechanisms yield the same consumption level. And, for larger values of $\lambda$, aggregate consumption is greater under exams than under markets.

It is interesting to note that, as indicated by the values of consumption in the tables below, exams under borrowing constraints can dominate exams under perfect capital markets in terms of aggregate consumption, even while being inferior to markets with borrowing constraints.[19]

[19] For the linear production technology chosen here, consumption, waste, and output were all monotonic functions of $\lambda$ and onsumption under exams with borrowing constraints was always higher than with perfect capital markets. In general, this need not be the case.

Table 1*

Perfect Capital Markets

| | $\lambda$ | $a^*$ | $y^*$ | $c^* - W$ | $\varepsilon^*$ |
|---|---|---|---|---|---|
| Markets | - | 0.5 | 1.25 | 1.25 | - |
| Exams | 0.2 | 0.5 | 1.25 | 0.7868 | 0.4632 |
| | 3 | 0.5 | 1.25 | 1.0625 | 0.1875 |
| | 5.309 | 0.5 | 1.25 | 1.1398 | 0.1102 |
| | 10 | 0.5 | 1.25 | 1.1946 | 0.0554 |
| | 50 | 0.5 | 1.25 | 1.2398 | 0.0102 |

Table 2*

Borrowing Constraints

| | $\lambda$ | $\widehat{a}$ | $\widehat{y}$ | $\widehat{c} - W$ | $\widehat{\varepsilon}$ |
|---|---|---|---|---|---|
| Markets | - | 0.3486 | 1.1602 | 1.1602 | - |
| Exams | 0.2 | 0.3628 | 1.1843 | 0.8617 | 0.3226 |
| | 3 | 0.4467 | 1.2346 | 1.0982 | 0.1364 |
| | 5.309 | 0.4675 | 1.2436 | 1.1602 | 0.0834 |
| | 10 | 0.4828 | 1.2482 | 1.2044 | 0.0438 |
| | 50 | 0.4966 | 1.2499 | 1.2415 | 0.0084 |

*The calculations above are for $H = 3$, $L = 1$ (so $z = 2$), $\overline{w} = 3$,and $\alpha = 0.5$. In order to allow easier comparisons between output and consumption, consumption is net of aggregate wealth.

## 7. Other Mechanisms

In this section we examine the implications of different allocative mechanisms all under the assumption that the market for loans is inoperative.[20]

[20] We have chosen not to specify the particular mechanism that closes down borrowing against human capital. This has the benefit of not introducing additional assumptions or details of the model that do not bear directly on the main point of this paper. The corresponding drawback is an inability to rigorously discuss interventions such as subsidized loans or other policies that would be likely to interact with the microfoundations for the inability to borrow.

### 7.1. Taxation

A very simple mechanism that ensures matching efficiency is to redistribute first-stage wealth (endowments) so that all individuals end up with equal endowments.[21] This extreme scheme guarantees efficient matching since the market (or for that matter, the test technology) can no longer discriminate among individuals based on their ability to pay, but rather solely on their willingness to pay. There are several valid objections, however, that can be raised to such a proposal. First, in the absence of a dynamic model, taxation of wealth erroneously appears to be an entirely innocuous instrument; it causes no distortions and hence maximizes aggregate consumption. In a more sophisticated model, with savings and effort decisions, this would not be the case.[22] Second, if wealth is not observable, the government is unable to tax it effectively.

Consider instead the taxation of the income individuals obtaining from schooling. This tax will have incentive effects, as will be seen shortly, but in a direction that improves matching efficiency. It will, again artificially, not have other disincentive effects due to the simple and static nature of the model (e.g. no disutility from labor, etc.). Thus, one way to think about the results is that they yield an upper bound for how large the potential distortions due to taxation would have to be in order to outweigh its benefits. Given these caveats, I do not solve for the optimal tax schedule but restrict attention to the effects of a proportional income tax.[23]

The effect of a proportional tax is to decrease the net gain from obtaining a high-quality education from $\Delta(a)$ to $(1-t)\Delta(a) \equiv (1-t)[X(a,H)-X(a,L)]$. Consequently, equilibrium

[21] Complete redistribution is not necessary, in general, to obtain the first best allocation. It is sufficient to ensure that all agents can afford $P_H^*$. Of course, the latter may not be feasible (i.e., $W$ may be smaller than $P_H^*$).

[22] See De Fraja (2002) for an analysis of the optimal income tax with heterogeneous agents and borrowing constraints with public and private schools.

[23] A proportional tax on all income would be unlikely to be optimal since a similar outcome could be obtained without taxing those individuals who attend $L$. This would possess the same positive matching incentives as a proportional tax on all individuals, but would have the additional benefit of allowing a lower tax on $H$ to obtain the same effects and no tax on those who attend $L$.

condition (3.5) must be modified to:

$$(1-t)\Delta(\widehat{a}) = \widehat{P}_H \tag{7.1}$$

with (3.6) and (3.7) remaining unchanged.

The decreased attractiveness of higher-quality education lowers the demand for the latter, decreasing its price and thus making it more affordable to higher-ability but poorer individuals but less attractive to lower-ability individuals. Consequently, as shown in Figure 4, the equilibrium value of $\widehat{a}$ increases and $\widehat{P}_H$ decreases. Output is unambiguously higher as is, in the absence of other distortions, aggregate consumption.

The effect of taxation under an exam mechanism is similar to its effect under market prices. With an exam mechanism, though, taxation has the additional benefit of decreasing wasteful expenditures, both because it decreases the spread of ability levels of the individuals that attend $H$, and hence the total level of expenditures associated with any given score, but also because the score associated with $H$ falls.

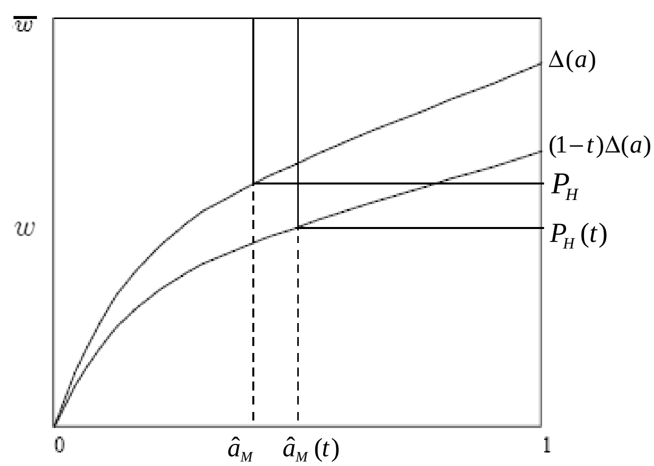


Figure 4

### 7.2. A Fellowship Scheme[24]

Suppose now that initial endowments of wealth are perfectly observable (but that it is still impossible or very costly to transfer these among individuals). We maintain, however, the assumption that the government is unable to observe exam expenditures. This modification does not affect the equilibria derived previously but it permits the government to implement a scheme whereby it maintains the market mechanism while bestowing "fellowships" on the basis of wealth and exam results.

The introduction of a fellowship scheme is modelled in the following way. In the first stage, the government announces the fellowship rule. This consists of an exam and a score $v_f$ that must be achieved in order to obtain a fellowship amount $m(w)$. In the next stage, schools set prices and individuals choose whether to take the exam or to simply pay the price associated with either the high or low quality school. Individuals who obtain fellowships must attend $H$.

There is a fellowship policy that comes arbitrarily close to achieving the first best level of output and consumption: in equilibrium the high-quality schools charge a price arbitrarily close $P_H^*$, the assignment of individuals to schools is arbitrarily close to the first best assignment, and wasteful expenditures are arbitrarily close to zero.

Below I describe the properties of the fellowship mechanism. For expositional simplicity I restrict attention to the case where $P_H^* \leq \overline{w}$, i.e., $\Delta(a^*) \leq \overline{w}$.[25]

Let the government's fellowship be given by a pair $v_f, \gamma$, with $\gamma > 0$, where

$$v_f = V(a^*, \gamma) \tag{7.2}$$

i.e., it is the score that an agent with ability $a^*$ would achieve with expenditures of $\gamma$. All

[24] I thank Sherwin Rosen for suggesting a long time ago that a mixed mechanism that combined both prices and exams might be optimal.

[25] The sole modification that must be made if $P_H^* > \overline{w}$ is for the government to place an upper bound on the fellowship that it will pay of $P_H^* - w + \epsilon$, otherwise schools have an incentive to charge a higher price knowing that the government, rather than individuals, will pay the incremental cost.

individuals who obtain a score of at least $v_f$ and whose wealth lies below $P_H$ are awarded a fellowship of the amount:

$$m(w) = P_H - w + \gamma \tag{7.3}$$

i.e., they pocket $\gamma$ and use the fellowship to attend the higher-quality school since it pays for the difference between $P_H$ and the individual's wealth.

When faced with this rule, all individuals with $\min(\Delta(a), w) \geq P_H$ will attend the high-quality school, and will pay in full in order to do so.[26] Furthermore, all individuals with $a \geq a^*$ and $w - E(a, v_f) + \gamma \leq \min(P_H, \Delta(a))$ will also attend the high-quality school with a fellowship. Note that agents with $a > a^*$ could afford to attend $H$ with a smaller fellowship. If exam expenditures are unobservable (a maintained assumption throughout), however, then the government is unable to keep the difference between $\gamma$ and $E(a, v_f)$. Of course, individuals with $a < a^*$ and $w - E(a, v_f) + \gamma \leq \min(P_H, \Delta(a))$ also desire a fellowship to attend $H$, but the fellowship amount is designed to be insufficient to allow them to do so since they would have to spend more than $\gamma$ in order to obtain $v_f$, leaving them with insufficient funds to afford $P_H$ even with a fellowship.

Note that the above discussion implies that in equilibrium $P_H$ will be strictly smaller than $P_H^*$ since if the price were at its perfect capital market level, the fraction of individuals willing and able to attend with this fellowship scheme would be smaller than $1 - \alpha$. Thus, in equilibrium we must have

$$P_H = \Delta(a') \tag{7.4}$$

where $a'$ is defined implicitly by

$$\int_{E(a,v_f)}^{\overline{w}} \int_{a^*}^{1} f(a, w)\ da\ dw + \int_{P_H}^{\overline{w}} \int_{a'}^{1} f(a, w)\ da\ dw = 1 - \alpha \tag{7.5}$$

[26]For the same reasons as described for the other mechanisms, $P_L = 0$. That is, in equilibrium there will be a measure of agents who attend $L$ whose wealth is arbitrarily close to zero, requiring a price of zero for the lower-quality school in order to clear markets.

Thus,

$$S(a,w) = \begin{cases} H, \ \forall a \geq a' \text{ and } w \geq E(a, v_f) \\ H, \ \forall a \in [a', a^*) \text{ and } w \geq P_H \\ L, \text{ otherwise} \end{cases} \tag{7.6}$$

Furthermore, exam expenditures by each agent, $e(a,w)$, are given by:

$$e(a,w) = \begin{cases} E(a, v_f), \ \forall \ a \geq a' \text{ and } w \in [E(a, v_f), P_H) \\ 0, \text{ otherwise} \end{cases} \tag{7.7}$$

yielding total waste of:

$$\varepsilon = \int_{a'}^{1} \int_{E(a,v_f)}^{\Delta(a')} E(a, v_f) f(a,w) \ dwda \tag{7.8}$$

The equilibrium is depicted in Figure 5. All agents with $a \geq a^*$ who can afford to attend $H$ either by paying the full price or via a fellowship, do so, i.e., all those with $w \geq E(a, v_f)$ In addition, those agents with $a \in [a', a^*)$, who can afford to attend $H$ without a fellowship also do so. All other agents attend $L$. The set of agents attending $H$ using fellowships is depicted in Figure 8 by the vertically shaded area. Those paying full price lie in the diagonally shaded area.

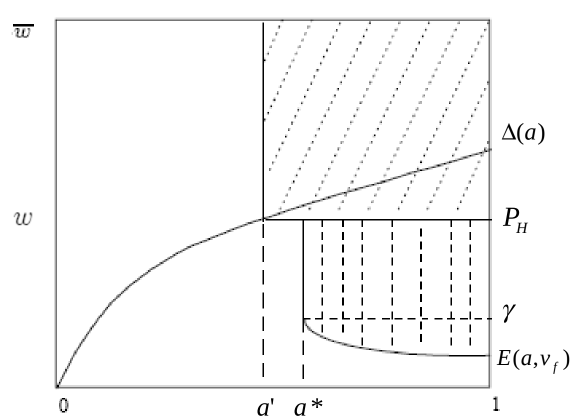


Figure 5

Note that the smaller is $\gamma$, the closer $a'$ is to $a^*$ and that the distance between these two can be made arbitrarily small by making $\gamma$ sufficiently small. Hence, as $\gamma \rightarrow 0$, the allocation tends to the efficient one and $P_H \rightarrow P_H^*$. Furthermore, as $\gamma \rightarrow 0$, $v_f$ does likewise, implying that aggregate waste also tends to zero. Thus, for a small enough $\gamma$, the allocation of individuals to schools (and hence output), and aggregate consumption, comes arbitrarily close to the first best level, $Y^*$ and $C^*$.

It is interesting to observe that the above fellowship scheme which maximizes both output and aggregate consumption does not require agents to obtain the highest score compatible with their wealth and some minimum ability level, say $a^*$. In fact, the score that they are required to obtain is independent of their wealth, despite the fact that the latter is observable. To see why, consider a fellowship scheme that requires individuals to obtain the score $\overline{v}_f(w)$ defined by $E(a^*, \overline{v}_f(w)) = w$ in order to obtain a fellowship of $m(w) = P_H^* - E(a^*, \overline{v}_f(w))$ for $w < P_H^*$. This policy will indeed deliver the first-best allocation of individuals to schools. However, unlike the previous fellowship mechanism, this arrangement does not maximize aggregate consumption as it results in a large amount of wasteful expenditures. Whereas in the preceding fellowship scheme individuals were spending $\gamma$ and below, under this alternative setup individuals whose wealth lies below $P_H^*$ must spend close to their entire wealth generating the fellowship score. Furthermore, this policy would require the government to raise a larger quantity of revenue since individuals receive as fellowships the difference between $P_H^*$ and $w - E(a, \overline{v}_f(w))$ rather than (at most) the difference between $P_H^*$ and $w - E(a, v_f)$.

## 8. Conclusion

This paper examined the properties of exams and markets as alternative allocation mechanisms in the presence of borrowing constraints. Exams are shown to dominate markets in terms of matching efficiency. Which mechanism delivers higher aggregate consumption depends on the power of the exam technology; for a sufficiently powerful test, exams dominate markets in terms of aggregate consumption as well. The effects of income taxation are analyzed and the optimal allocation mechanism when wealth is observable is derived. The latter consists of a fellowship scheme in which markets set school prices but the government gives out fellowships based on need and the ability to obtain a given exam score.

This paper can be considered a first step in thinking about why countries use different mixes of prices and exams to guide admission decisions. For example, if wealth (or income) is easily observable (something that one may think more likely in an advanced economy with a sophisticated taxing and auditing system than in a less developed country), then a fellowship scheme based on need and exam scores will tend to be a more efficient allocation mechanism than either prices or exams used on their own. Thus, ceteris paribus, one might expect to see developed countries relying more on fellowships and prices (with private ownership of schools) and developing countries relying more on exams and public ownership of schools. The latter to a large extent is true, with most developing countries characterized by public universities, low fees, and exams to guide admission decisions. The characterization of advanced economies, however, is more heterogenous, with some countries such as Japan relying on exams at virtually every stage of the educational process whereas others use a mixture of exams and prices. Similarly, one might expect to see countries in which borrowing constraints against human capital are less binding relying on a price rather than an exam mechanism.

There are many additional questions that suggest themselves for future research. One potential drawback of the analysis presented is that the distribution of schools is exogenous. In a general equilibrium framework, however, this distribution would in general also depend on the allocation mechanism used. This would require specifying the objective function of schools which is non-trivial as most schools are non-profit institutions with a variety of actors with potentially quite different objectives playing important roles in decision making, e.g. principles, trustees, teachers, parents, and local and state government.[27] Other

[27] See Rothschild and White (1993) for an insightful discussion of the application of economic analysis to the study of university behavior. See Epple and Romano (1998) for a model with a (passive) public school sector and a competitive profit-maximizing private school sector.

avenues for future research include incorporating peer effects into the analysis, as in Epple and Romano (1998), and analyzing how different levels of education may call for different allocation mechanisms. If, for example, human capital accumulation prior to tertiary level is important in determining future outcomes, then ensuring that access to high quality primary and secondary education be independent of income (though not necessarily independently of ability) may be much more important than interventions at the tertiary level. Extending the analysis to an imperfectly competitive framework would also be valuable to understand, in particular, the functioning of tertiary institutions in the US. Lastly, there are undoubtedly questions of political economy that guide the use of different allocation schemes as well as considerations of efficiency (which may help to explain why we see public universities subsidized by the general tax payer). Note, for example, that individuals that attend high quality schools in the presence of borrowing constraints are actually better off than under perfect capital markets since borrowing constraints lower the price of the high-quality school or the score required for admission. Similarly, there will be a group of agents that prefers a less powerful test technology than a more powerful one since the former allows wealth to play a greater role relative to ability.

## References


[1] Acemoglu, Daron, "Matching, Heterogeneity and the Evolution of Income Distribution," *Journal of Economic Growth*, March 1997, vol 2, pp 61-92.

[2] Becker, Gary, "A Theory of Marriage: Part I," *Journal of Political Economy*, 813-846, 1973.

[3] Becker, Gary and Nigel Tomes, "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility," *Journal of Political Economy,* 87(6), 1153-1189, 1979.

[4] Benabou, Roland, "Heterogeneity, Stratification and Growth," *American Economic Review*, 86(3), 84-609, 1996.

[5] Burdett, Ken and Melvyn Coles, "Marriage and Class," *Quarterly Journal of Economics,* 112(1), 141-168, 1997.

[6] Cole, Harold, Mailath, George and Andrew Postlewaite, "Social Norms, Savings Behavior, and Growth," *Journal of Political Economy*, 1092-1125, 1992.

[7] De Fraja, Gianni, "The Design of Optimal Education Policies," *Review of Economic Studies,* 69(2), 437-466, 2002.

[8] Epple, Dennis and Richard Romano, "Competition between Private and Public Schools, Vouchers, and Peer Group Effects," *American Economic Review*, 88, 33-62, 1998.

[9] Evans, David and Boyan Jovanovic, "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *Journal of Political Economy*, 97(4), 808-827, 1989.

[10] Fernández, Raquel and Jordi Gali, ""To Each According to ...? Tournaments, Markets and the Matching Problem Under Borrowing Constraints," *Review of Economic Studies*, 66 (4), 799-824, 1999.

[11] Fernández, Raquel and Richard Rogerson, "Income Distribution, Communities and the Quality of Public Education," *Quarterly Journal of Economics*, 111(1), 135-164, 1996.

[12] Fernández, Raquel and Richard Rogerson, "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education Finance Reform," *American Economic Review*, 1998.

[13] Fernández, Raquel and Richard Rogerson, "Equity and Resources: An Analysis of Education Finance Systems," *Journal of Political Economy*, 111(4), 858-897, 2003.

[14] Freeman, Scott, "Equilibrium Income Inequality Among Identical Agents," *Journal of Political Economy*, 104, no. 5, 1047-1064, 1996.

[15] Galor, Oded and Joseph Zeira, "Income Distribution and Macroeconomics," *Review of Economic Studies*, 35-52, 1993.

[16] Glomm, Gerhard, and B. Ravikumar, "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy*, 100(4), 818-834, 1992.

[17] Kremer, Michael and Eric Maskin, "Market Segregation by skill and the Rise in Inequality," mimeo, Harvard, 1996.

[18] Legros, Patrick and Andrew Newman, "Monotone Matching in Perfect and Imperfect Worlds," *Review of Economic Studies*, 2002.

[19] Loury, Glenn, "Intergenerational Transfers and the Distribution of Earnings," *Econometrica*, 843-867, 1981.

[20] Pratap, Sangeeta and Silvio Rendon, "Firm Investment under Imperfect Capital Markets: A Structural Estimation," *Review of Economic Dynamics*, 2003, 6,3, pp. 513-545.

[21] Rothschild, Michael and Lawrence J. White, "The University in the Marketplace: Some Insights and Some Puzzles," in C. Clotfelter and M. Rothschild, eds., *Studies in Supply and Demand in Higher Education*, Chicago: University of Chicago Press, 1993.

[22] Rothschild, Michael and Lawrence J. White, "The Analytics of the Pricing of Higher Education and Other Services in Which the Customers are Inputs," *Journal of Political Economy*, 103(3), 573-586, 1995.

[23] Rothschild, Michael and Joseph E. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90, 629-650, 1976.

[24] Sattinger, Michael, "Assignment Models and the Distribution of Earnings," *Journal of Economic Literature*, 31, 831-880, 1993.